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## **EDITORIAL**

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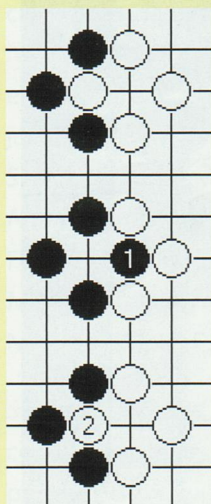
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## HIKARU NO GO: THE MATHS MOVES

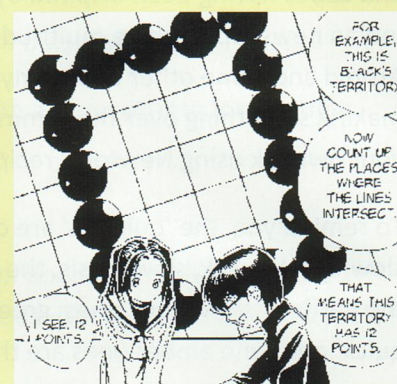
Hikaru no Go (The Go of Hikaru) is a very successful Japanese graphic novel series, a collection of 23 manga books written by Yumi Hotta and illustrated by Takeshi Obata. The books tell the story of a sixth grade boy, Hikaru Shindo, who discovers a Go board haunted by the spirit of an ancient Go master, Fujiwara-no-Sai. Hikaru will learn the rules of Go, then discover a passion for the game, and eventually decide to study this strategy game as a career and become a professional Go player. Playing Go is serious business in Japan, and a professional 5-dan Go player, Yukari Umezawa, has supervised the technical details in the story. The Hikaru no Go manga series has known a huge success in Japan and all over the world, dramatically increasing the popularity of the Go game, especially among youngsters.

The Asian game of Go is a board game with an incredible amount of mathematical content. The young readers of the Hikaru no Go manga series are thus unwittingly introduced to all kinds of mathematical topics related to the game of Go. The game of Go is played on a 19 by 19 grid, with players alternatively placing black or white lens shaped pieces (the stones) on the intersections of the grid lines (the points). Once placed on the board the stones may not be moved, unless captured by the enemy. The goal of the game of Go is to surround as many empty points and to capture as many stones as possible. The player with the most territory points and prisoners wins. Only two simple rules are needed to describe the game of Go. The first rule, the liberty rule, states that a stone or a formation (a group of adjacent stones connected by horizontal or vertical grid segments) needs at least one adjacent empty point (a liberty) to stay on the board. Once a stone or a formation loses its last liberty, it is captured and removed from the board.

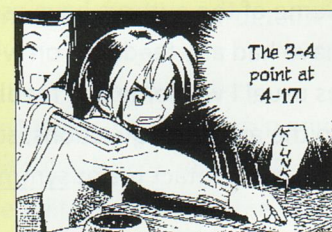


The second rule, the rule of ko, states that one cannot capture a stone if this will result in the same game configuration that was on the board before the last two moves were played. This rule is needed to prevent the game from entering a never-ending cycle. The rule of ko basically states that after the capture of a stone, in a situation like the one in the diagram, the other player cannot recapture right away the lost territory point. Therefore white, in the given example, cannot move at 2 immediately after the black move at 1. If the sequence of moves 1 and 2 were allowed, then the game would go through an endless alternation of identical board configurations.

A basic mathematical concept involved in the game of Go is the concept of area. Area is obtained by counting unit squares or, in the case of a rectangular shape, by multiplication. In Go the empty territory point takes the role of the unit square. A nice example of this concept of area is presented in Hikaru no Go in volume 3, as part of a Go tutorial given by one kid to another kid. As explained in the text, if a territory has an irregular shape, by moving stones one can produce a rectangular shape with the same number of empty territory points, for easier counting.



Another basic mathematical concept used in the game of Go is the concept of Cartesian coordinates. Two coordinates give the position of each stone on the Go board, and the board itself, with its 19 by 19 grid, very much resembles a coordinate plane. This is how Sai, the Go playing spirit, is able to play Go with the help of Hikaru. The ghost cannot place stones on the Go board, but he can tell Hikaru, who is the only one to hear his voice, the coordinates of the Go moves that Hikaru should make.

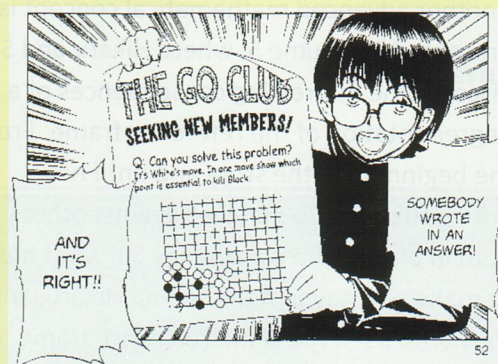




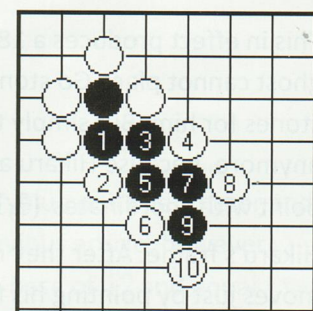
As it turns out, the ordered pair used to describe the position of a point on the Go board is not the usual  $(x, y)$  ordered pair of algebra, but a reversed  $(y, x)$  ordered pair, with a reference y-axis pointing down. Thus the upper left corner of the Go board has coordinates  $(1, 1)$  and the lower right corner has coordinates  $(19, 19)$ .

The game of Go, like the game of Chess, provides ample occasions for deductive reasoning. This is especially true when solving Go problems, which, like mathematical problems, may require the evaluation of multiple if-then statements, the exploration of the game tree, and the discovery of an optimal path, a sequence of moves that will take the game from the current board configuration to the desired outcome.

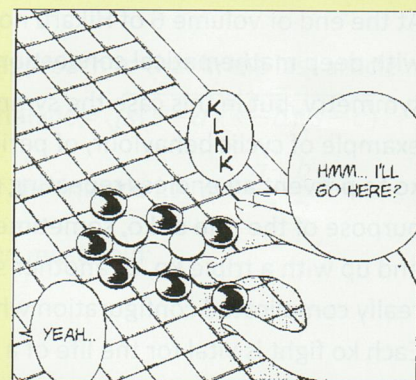
A good example of a Go life-and-death problem is presented in volume 3. When Hikaru and his friends decide to advertise the need for more members to join their Go club, they post a Go problem. Their strategy works; someone shows interest and finds the solution. When solving a mathematical problem a similar approach is required to develop a procedure, a sequence of steps that will get us from the known quantities to the unknown ones, to the answer.



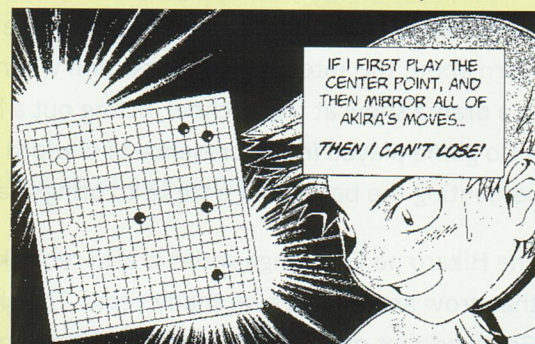
The game of Go also presents an occasion for inductive reasoning. The analysis of a repeating pattern is necessary when dealing with a Go ladder. Although this formation looks more like a staircase, we still call it a ladder, probably because of an initially inaccurate translation. Understanding Go ladders and being able to extend the pattern, both on the board and during a mental analysis of the situation, is one of the most important Go techniques that every novice Go player needs to learn. As seen in the diagram, the black formation has only one liberty left. By placing white stones alternatively on one side and on the other side of the ladder, the white player is able to keep the black formation in this precarious state, with just one liberty left. It is quite interesting to discover that after four moves, two black moves and two white moves, the ladder configuration on the board looks exactly like what it was before, only extended a little bit diagonally. This periodic pattern will repeat itself, and the ladder will grow longer. The black formation cannot escape capture unless the ladder hits a friendly black stone. Otherwise white will capture the black stones when the ladder hits the edge of the board or another white stone.



In Hikaru no Go the ladder formation is not discussed in any detail, but only briefly shown in volume 4 as part of a teaching game. The game of Go, if played consistently, can help someone develop and improve pattern identification skills. This is also very useful when solving some mathematical problems. As a side note, in the game of Chess we also have a situation that requires inductive reasoning, the endgame with a King and a Rook.

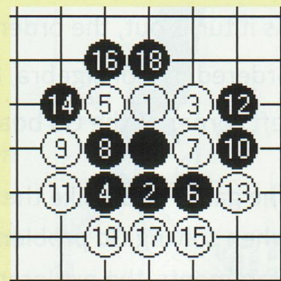


The game of Go also presents occasions to apply more advanced mathematical concepts, like, for example, the concept of symmetry. In Hikaru no Go in volume 2 we witness a game of Go between two players of very different strengths. The weaker player decides to mirror the moves of the stronger player, with the unsupported belief that in this way he cannot lose. The centre of the board is in this case becomes a centre of symmetry. The symmetry operation involved is spatial inversion coupled with a change of colour. This strategy, unfortunately, does not work.

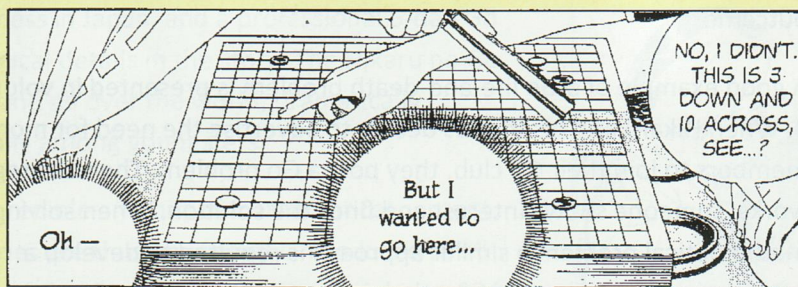




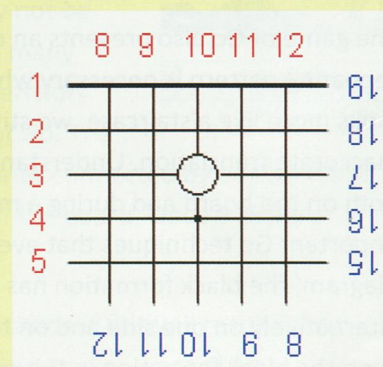
By playing as indicated in the diagram white will end up capturing the group of five black stones in the centre. This results in a loss of symmetry on the Go board and a loss of territory and stones for the black player. Good Go players memorise such standard sequences of moves, the same way good Chess players memorise Chess opening. This memory training exercise is an excellent workout for the brain. These Go and Chess players can then enjoy the educational benefits of an improved memory, with better learning outcomes and superior academic accomplishments.



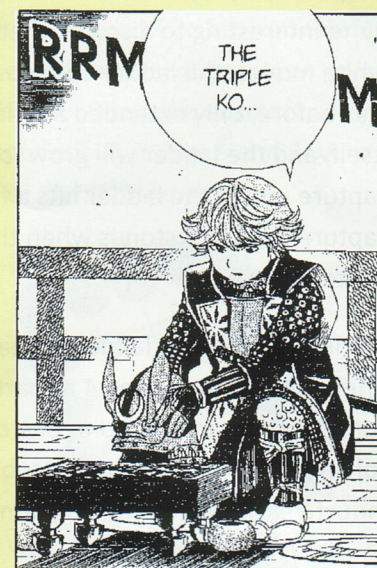
Another advanced mathematical concept is presented in volume 5, where Hikaru and Sai discover the interesting consequences of a 180 degree rotation of the reference frame. From the beginning of the story and until now, during Hikaru's Go matches against other human players, he was told by Sai what moves to make. Sai stayed right behind Hikaru, and they both used the same reference frame, the same coordinate axes. But now Hikaru wants to play against Sai, and he asks the spirit to move in front of him, to take the place of a human opponent.



This in effect produces a 180 degrees rotation of Sai's reference frame. Since the ghost cannot place Go stones on the board, he has to tell Hikaru where to place stones for him. But simply telling Hikaru the Cartesian coordinates does not work anymore, because Hikaru and Sai are now using different reference frames. A point with coordinates (3,10) in Sai's frame now has coordinates (17,10) in Hikaru's frame. After they realize this puzzling fact Sai decides to indicate his moves just by pointing his fan at the respective points on the board.



At the end of volume 6 of Hikaru no Go we learn about a very special Go situation with deep mathematical connections, the triple ko. This is another example of symmetry, but in this case the symmetry is not in space, it is in time. It is an example of cyclic behaviour, of periodicity. The game of Go has the simple rule of ko to prevent an endless repeating of the same moves. However, defeating the purpose of the rule of ko, sometimes, in extremely rare occasions, a Go game will end up with a triple ko, or another special cyclic situation. The triple ko requires a really complicated configuration where three ko fights appear at the same time. Each ko fight is vital for the life of a big formation, and neither player can afford to lose any of the ko fights. In a triple ko situation the Go board configuration repeats itself after every six moves, three white moves and three black moves. The professional Go players, of course, recognize the triple ko situation and stop the game. Because neither player wins, the triple ko is considered an ill omen. We learn about this interesting superstition when Hikaru and his classmates take part in a drama show at their school, acting out a historical event in which a triple ko was recorded. It was in 1582 during a Go game played in the presence of warlord Nobunaga. The next day Nobunaga was killed by a rebelled ally, cementing the belief that a triple ko brings bad luck.



The Hikaru no Go manga series is also remarkable because, unlike most other graphic novels, it presents characters that grow and evolve over many years. The children become teenagers, almost eighteen years old. The learning of Go takes a lot of effort, the pursuit of mastery is spread over many years, and this is a message that also applies to the learning of mathematics, or other subjects.



By promoting this message of hard work and dedication the Hikaru no Go story also delivers a very effective motivational discourse. Even more, some concrete advice given to Hikaru to help him improve his performance during Go games has general value and also applies to other fields of study, or qualifies as an exam taking strategy. The readers learn about the need to stay calm, the need to clear the mind of external distractions, the benefits of using intuition, the value of perseverance, the benefits of being part of a study group, and they also realize the never-ending nature of the learning path. In this way the Hikaru no Go graphic novel is offering our youngsters an authentic learning philosophy.

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**Călin Galeriu**